

Topic #9

AB/BC

Miscellaneous

2000 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

5. Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

(b) Find all points on the curve whose x -coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the x -coordinate of each point on the curve where the tangent line is vertical.



AP Calculus AB
2001 Free-Response Questions

5. A cubic polynomial function f is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

where a , b , and k are constants. The function f has a local minimum at $x = -1$, and the graph of f has a point of inflection at $x = -2$.

- (a) Find the values of a and b .

- (b) If $\int_0^1 f(x) dx = 32$, what is the value of k ?

END OF EXAMINATION

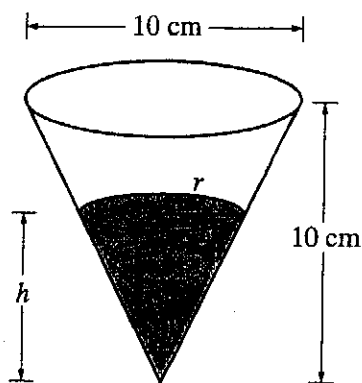
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2002 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

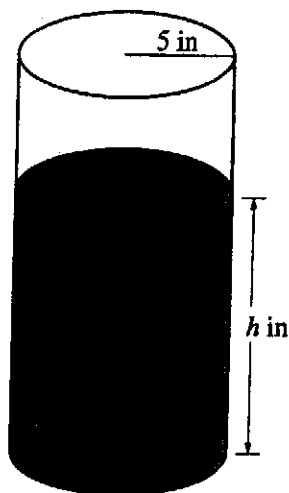


5. A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $-\frac{3}{10}$ cm/hr.

(Note: The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

- Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.
- Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.
- Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

2003 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS



5. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t , measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

(a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

(b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for h as a function of t .

(c) At what time t is the coffeepot empty?

6. Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ 5-x & \text{for } 3 < x \leq 5. \end{cases}$$

- (a) Is f continuous at $x = 3$? Explain why or why not.
 (b) Find the average value of $f(x)$ on the closed interval $0 \leq x \leq 5$.
 (c) Suppose the function g is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ mx+2 & \text{for } 3 < x \leq 5, \end{cases}$$

where k and m are constants. If g is differentiable at $x = 3$, what are the values of k and m ?

END OF EXAMINATION

2004 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

**CALCULUS AB
SECTION II, Part B**

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

4. Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

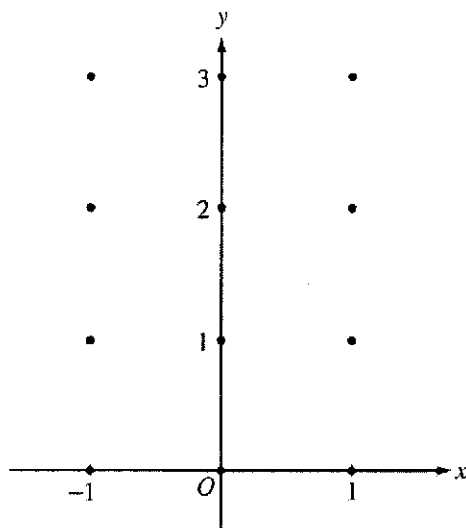
(a) Show that $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$.

(b) Show that there is a point P with x -coordinate 3 at which the line tangent to the curve at P is horizontal.
Find the y -coordinate of P .

(c) Find the value of $\frac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point P ? Justify your answer.

2004 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

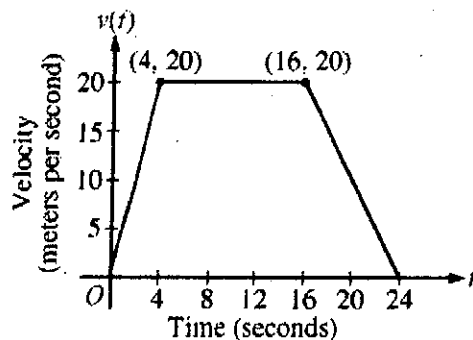
6. Consider the differential equation $\frac{dy}{dx} = x^2(y - 1)$.
- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the pink test booklet.)



- (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy -plane. Describe all points in the xy -plane for which the slopes are positive.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.
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END OF EXAMINATION

2005 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS



5. A car is traveling on a straight road. For $0 \leq t \leq 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.
- Find $\int_0^{24} v(t) \, dt$. Using correct units, explain the meaning of $\int_0^{24} v(t) \, dt$.
 - For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.
 - Let $a(t)$ be the car's acceleration at time t , in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.
 - Find the average rate of change of v over the interval $8 \leq t \leq 20$. Does the Mean Value Theorem guarantee a value of c , for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?
-

WRITE ALL WORK IN THE TEST BOOKLET.

2006 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

6. The twice-differentiable function f is defined for all real numbers and satisfies the following conditions:

$$f(0) = 2, \quad f'(0) = -4, \quad \text{and} \quad f''(0) = 3.$$

- (a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a . Show the work that leads to your answers.
- (b) The function h is given by $h(x) = \cos(kx)f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.
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WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM

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2007 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

3. The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x . The function h is given by $h(x) = f(g(x)) - 6$.
- (a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.
- (b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.
- (c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t) dt$. Find the value of $w'(3)$.
- (d) If g^{-1} is the inverse function of g , write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.
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WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF PART A OF SECTION II

2008 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

2. Concert tickets went on sale at noon ($t = 0$) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \leq t \leq 9$. Values of $L(t)$ at various times t are shown in the table above.
- (a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. ($t = 5.5$). Show the computations that lead to your answer. Indicate units of measure.
- (b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.
- (c) For $0 \leq t \leq 9$, what is the fewest number of times at which $L'(t)$ must equal 0? Give a reason for your answer.
- (d) The rate at which tickets were sold for $0 \leq t \leq 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. ($t = 3$), to the nearest whole number?
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WRITE ALL WORK IN THE PINK EXAM BOOKLET.